

**2003000204040057**  
**EXAMINATION FEBRUARY-MARCH 2024**  
**BACHELOR OF SCIENCE (FOURTH SEMESTER)**  
**MATHEMATICAL MODELING LEVEL 4**

[Time: As Per Schedule]

[Max. Marks: 50]

**Instructions:**

1. **1. Fill up strictly the following details on your answer book**
  - a. Name of the Examination : **BACHELOR OF SCIENCE (FOURTH SEMESTER)**
  - b. Name of the Subject : **MATHEMATICAL MODELING LEVEL 4**
  - c. Subject Code No : **2003000204040057**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. First question carry 8 marks and other carries 14 marks.
6. Follow usual notations.
7. Use scientific non programmable calculator.

Seat No:

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Student's Signature

**Q.1 Answer the following: (Any Four)**

**8**

- 1) Let  $x(t) = x(0)e^{2t}$ . If  $x(t) = 2$  then find  $x(0)$ .
- 2) Let  $x(T) = x(0) \exp(aT)$ . If the population  $x(T)$  at any time  $T$  will double its present size  $x(0)$  then find  $T$ .
- 3) If  $\frac{d^2x}{dt^2} = a - 2bx$  then find the condition at which  $\frac{d^2x}{dt^2} = 0$  and  $\frac{d^2x}{dt^2} > 0$ .
- 4) Differentiate  $f(x, y, a) = 0$ ;  $a$  is parameter partially with respect to  $x$ .
- 5) Find the Orthogonal trajectories of the family of curves  $y = -x$ .
- 6) Find the curves for which the projection of the normal on the  $X$  -axis is of constant length.

**Q.2 Answer the following: (Any Two)****14**

- a) Obtain population growth model and solve it. Find the time at which the population will become half of its present size.
- b) State the population growth model. In an archaeological wooden specimen, only 25% of original radio carbon-12 present. Find the time when was it made?
- c) State the model of growth of Mathematics  $M(t)$ . In 1900 year, the amount of Mathematics is 1000. If the doubling period is 10 year then Estimate the number of amounts of mathematics in the year 1940.

**Q.3 Answer the following: (Any Two)****14**

- a) Cigarette consumption in a country increased from 50 per capita in 1900 AD to 3900 per capita in 1960 AD. Assuming that the growth in consumption follows a logistic law with a limiting consumption of 4000 per capita, estimate the consumption per capita in 1950 AD.
- b) Obtain the model of spread of Infectious diseases. Prove that after very long time all persons will be affected.
- c) Find the solution of a generalization of the logistic model

$$\frac{1}{N} \frac{dN}{dt} = \frac{r}{\alpha} \left( 1 - \left( \frac{N}{K} \right)^\alpha \right), \alpha > 0$$

**Q.4 Answer the following: (Any Two)****14**

- a) Find the Orthogonal trajectories of the family of curves  $x^2 + y^2 - 2ax = 0$ .
- b) (i) Obtain the Orthogonal trajectories of the family of curves  $r = a(1 - \cos\theta)$ .  
(ii) Find curves for which the projection of the normal on the x-axis is of constant length.
- c) Find the Orthogonal trajectories of the family of Confocal conics  $\frac{x^2}{a^2 + \beta} + \frac{y^2}{b^2 + \beta} = 1$ , where  $\beta$  is a parameter.

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